

On moments of inertia of strongly deformed odd- A nuclei.

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With 7 figures in the text

Low energy rotational spectra have been observed experimentally for nuclei in the mass regions $A \sim 25$, $150 < A < 190$ and $A > 225$. From the type of rotational spectra exhibited, one may conclude that these nuclei possess rotational symmetry around the axis of deformation and furthermore also reflection symmetry with respect to a plane perpendicular to the axis of deformation.

For the nuclei in these mass regions, the frequency of rotation is low compared to the frequency of single-particle excitation. The adiabatic coupling scheme (A. Bohr, Dan. Mat. Fys. Medd. 26, no. 14, 1952, A. Bohr and B. R. Mottelson, Dan. Mat. Fys. Medd. 27, no. 16, 1953) is then applicable. The nucleus shows collective rotational motion as a whole, while its shape is preserved. The intrinsic motion of the individual nucleons is then conveniently expressed with respect to the deformed nuclear field, which in this first approximation is considered as static. The effects of the Coriolis and centrifugal forces is thus neglected in this adiabatic approximation in calculating the intrinsic wave functions.

One may now first try to describe the intrinsic motion as a completely independent particle motion in a shell model potential, somewhat generalized to include the effect of deformation (S. Moszkowski, Phys. Rev. 99, 303, 1955, S. G. Nilsson, Dan. Mat. Fys. Medd. 29, no. 16, 1955, K. Gottfried, Phys. Rev. 103, 1017, 1956). In such a potential, each intrinsic orbital may be labelled by the constant of motion Ω_i , which represents the projection of the single-particle angular momentum along the symmetry axis. Such an orbital is doubly degenerate, corresponding to the rotational symmetry around the nuclear axis of deformation. The ground state of an even-even nucleus corresponds to the single-particle states being filled with two nucleons in each orbital with a total $\Omega = \sum \Omega_i = 0$. For an odd- A nucleus the total angular momentum component Ω along the symmetry axis equals Ω_i of the last odd nucleon.

Many properties of an odd- A nucleus are mainly determined by the last odd particle. Calculated magnetic moments and rotational decoupling factors based on the single particle wave functions (Nilsson, Gottfried) as well as a successful classification of beta and gamma transitions in terms of the quantum numbers of these wave functions (see e.g. G. Alaga, Phys. Rev. 101, 432, 1955, B. R. Mottelson and S. G. Nilsson, Dan. Mat. Fys. Medd., forthcoming) demonstrate the applicability of the coupling scheme and the somewhat astonishing accuracy of the calculated wave functions of the last odd particle.

It is now tempting to calculate also the moment of inertia associated with the experimentally encountered rotational band. This involves estimating the

coupling between the single-particle motion and the collective rotational motion due to the Coriolis force (neglected in the lowest approximation). However, as has been pointed out by A Bohr and B. R. Mottelson (Dan. Mat. Fys. Medd. 30, no 1, 1955), any independent particle wave function always renders the rigid moment of inertia, which is of the order of twice what is experimentally found for even-even nuclei in this region. This deviation is generally understood as an effect of the correlation between the pairs of nucleons moving in the doubly degenerate orbitals (Bohr, Mottelson, and Pines, 1958, in press). That there is a strong correlation in the motion of a pair of nucleons is also indicated from the fact that the low lying intrinsic excitations in the even-even nuclei are found to possess a much higher energy than those in the odd nuclei. No excited states in even-even nuclei have been found below 1 MeV in the mentioned regions of nuclei and the pairing energy may be estimated to be about 2 MeV.

The part of the intrinsic wave functions involving the paired nucleons is thus less well understood than that associated with the last odd nucleon. At present, one cannot calculate the moment of inertia in more detail¹ (cf. Bohr-Mottelson, 1955). However, if the experimental moment of inertia, associated with the collective rotational motion, of an odd-A nucleus is compared with that of the preceding even-even nucleus of mass number $A-1$, the difference in moment of inertia may be considered as the isolated contribution of the last odd nucleon whose wave function is more accurately known. Such a systematic difference is experimentally encountered and found to be of the order of 20 % of the even-even value.

It is the aim of the present paper to make a comparison between this experimentally found difference in the moments of inertia and the single particle contribution calculated on the basis of the wave functions given by S. G. Nilsson (Dan. Mat. Fys. Medd. 29, no. 16, 1955). This reference is denoted SGN in the following.

Calculation of the contribution of J^{orb} .

In the description of a deformed nucleus corresponding to the adiabatic coupling scheme, the wave function is given by

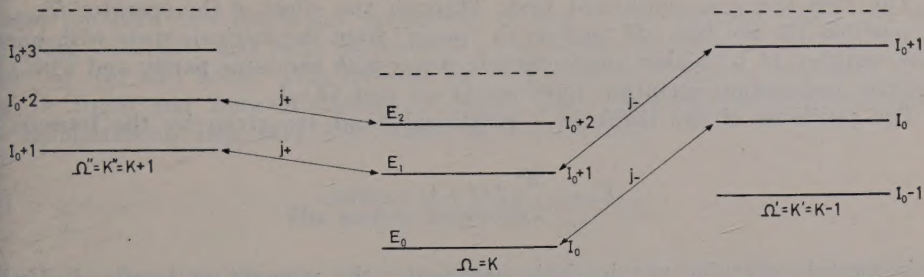
$$\Psi = \sqrt{\frac{2I+1}{16\pi^2}} \{D_{MK}^I \chi_{\Omega} + (-)^{I-j} D_{M-K}^I \chi_{-\Omega}\}. \quad (1)$$

Here I represents the total angular momentum operator and K its component along the nuclear symmetry axis. Due to the rotational symmetry of the system, the quantum number K is equal to the quantum number Ω .

The wave function χ_{Ω} describes the intrinsic motion of the nucleons in the deformed field and the D_{MK}^I the collective motion of the system as a whole. The second term of the wave function arises from the reflection symmetry mentioned above.

The total Hamiltonian operator is written in terms of the intrinsic coordinates referring to the principal axes of the system. If axis 3 is chosen as the

¹ Note added in proof: Lately the pair correlation functions based on the Bardeen model of superconductivity have been applied by S. T. Belyaev (Effect of Pairing Correlations in Nuclear Properties, 1958) to nuclei and appear to be able to account (at least qualitatively) for the moments of inertia of even-even nuclei.



The figure shows the coupling of the Coriolis force between levels based on different intrinsic structures ($\Omega = \frac{1}{2}$).

symmetry axis, the moments of inertia referring to the axes 1 and 2 are equal and are denoted J in the following.

$$J_1 = J_2 = J.$$

The separation of the wave function according to eq. (1) is correct only to lowest order in the frequency of rotation. In fact the rotational motion is coupled to that of the nucleons by a part of the total Hamiltonian operator arising from the neglected effect of the Coriolis force. The Coriolis coupling acts between all the particles and the collective rotation and gives the whole moment of inertia. For the last odd nucleon this coupling is given by

$$-\frac{\hbar^2}{2J} (I_+ j_- + I_- j_+),$$

where $I_{\pm} = I_1 \pm I_2$ and $j_{\pm} = j_1 \pm j_2$. The vector $\mathbf{j}$ (j_1, j_2, j_3) represents the angular momentum of the last odd nucleon.

This coupling, which is small, when the collective frequency is much smaller than the single-particle frequency, is neglected in first approximation. Thus when the energy of the system is calculated, only diagonal elements of this operator are taken into account (diagonal elements only occur for the case $\Omega = \frac{1}{2}$).

This operator is now considered as a perturbation

$$H_{\text{pert}} = -\frac{\hbar^2}{2J} (I_+ j_- + I_- j_+), \quad (2)$$

where J is the moment of inertia when the non-diagonal contribution of the operator is neglected. K and Ω are then good quantum numbers and the non diagonal matrix elements are given by

$$\left. \begin{aligned} \langle I' K' \Omega' | (I_+ j_- + I_- j_+) | I K \Omega \rangle &= \sqrt{(I+K)(I-K+1)} \delta_{I'I} \delta_{K'K-1} \langle f | j_- | i \rangle, \\ \langle I'' K'' \Omega'' | (I_+ j_- + I_- j_+) | I K \Omega \rangle &= \sqrt{(I-K)(I+K+1)} \delta_{I''I} \delta_{K''K+1} \langle f | j_+ | i \rangle. \end{aligned} \right\} \quad (3)$$

The initial and final states of the odd nucleon are denoted by i and f . The effect of H_{pert} is thus to couple states of different K (different rotational bands for which $K = \Omega$) but with the same I (see fig. above).

In this description the ground state of an odd- A nucleus corresponds to the single particle states (of nucleonic motion in the deformed field) being filled with two nucleons in each level up to the last odd nucleon.

The case $\Omega = \frac{1}{2}$ is considered first. Through the effect of the operator H_{pert} , it is possible for the last odd nucleon to "jump" from the intrinsic state with quantum number Ω to higher single-particle states with the same parity and with the angular momentum quantum numbers $\Omega + 1$ and $\Omega - 1$.

The positions of the levels of a rotational band are given by the formula

$$E_{\text{rot}} = \frac{\hbar^2}{2J} I(I+1) + \text{const.} \quad (4)$$

This formula contains one intrinsic parameter, the moment of inertia J . If the couplings are taken into account, the same formula (4) may still be used to describe the level ordering, the rotational parameter now being replaced by $J + \delta J$.

On account of the perturbation, the ground state E_0 and the first excited rotational state E_1 (see fig. above) are pressed down by

$$\left. \begin{aligned} \Delta E_0 &= (\sqrt{2I_0})^2 \left(\frac{\hbar^2}{2J} \right)^2 \sum_f \frac{|\langle f | j_- | i \rangle|^2}{E_i - E_f}, \\ \Delta E_1 &= (\sqrt{2(I_0 + 1)})^2 \left(\frac{\hbar^2}{2J} \right)^2 \sum_f \frac{|\langle f | j_+ | i \rangle|^2}{E_i - E_f} + (\sqrt{2(2I_0 + 1)})^2 \left(\frac{\hbar^2}{2J} \right)^2 \sum_f \frac{|\langle f | j_- | i \rangle|^2}{E_i - E_f} \end{aligned} \right\} \quad (5)$$

The rotational energies are neglected in the denominators and the energy differences are thus given by the single-particle energies E_i and E_f .

The following expression is then obtained for the contribution of the odd particle to the moment of inertia on account of excitations to higher levels

$$\delta J_A = \frac{\hbar^2}{2} \sum_f \frac{|\langle f | j_{\pm} | i \rangle|^2}{E_f - E_i}. \quad (E_f > E_i) \quad (6)$$

In addition to the coupling of the odd particle to higher levels, it must also be possible to get a contribution when pairs are broken in lower lying orbitals and again formed in the highest orbital populated by the last odd nucleon. A particle is thought to "jump" to the level Ω and to form a pair there. If no new pair were formed, the energy denominators in the second order perturbation theory should be great and the corresponding contribution to δJ very small. This is the reason for the smallness of the contribution of these processes in the even-even nuclei, which involve the same excitation of particles in lower lying orbitals into the state Ω .

For the contribution to the moment of inertia δJ_B due to the coupling to filled states $E_f < E_i$, eq. (6) holds with the indices i and f interchanged. The total inertial effect on account of the excitations considered is thus

$$\delta J^{\text{orb}} = \delta J_A + \delta J_B. \quad (7)$$

In the case $\Omega = \frac{1}{2}$, eq. (4) is modified and the rotational energies are obtained from

$$E_{\text{rot}} = \frac{\hbar^2}{2J} [I(I+1) + a(-)^{I+\frac{1}{2}}(I+\frac{1}{2})] + \text{const.} \quad (8)$$

There are here two rotational parameters J and a (the latter is called the decoupling constant) and both parameters are affected. However, an analysis shows that eq. (6) still holds. This formula for the moment of inertia was first given by D. Inglis and Bohr-Mottelson (D. Inglis, Phys. Rev. 97, 1955, A. Bohr and B. R. Mottelson, Dan. Mat. Fys. Medd. 30, No. 1, 1955).

The matrix elements $\langle f | j_{\pm} | i \rangle$

The wave functions for the single-particle states i and f are those calculated in reference SGN for essentially a nonisotropic harmonic oscillator potential with inclusion of a spin-orbit coupling. The normalized basic vectors employed in this reference are given by

$$|N\Omega\rangle = \sum_l \{A_{l\Omega-\frac{1}{2}} |Nl(\Omega-\frac{1}{2})+\rangle + A_{l\Omega+\frac{1}{2}} |Nl(\Omega+\frac{1}{2})-\rangle\}, \quad (9)$$

where N represents the total number of oscillator quanta and l , Λ and Σ are the quantum numbers of the operators l^2 , l_z and s_z , respectively.

$$\mathbf{j} = \mathbf{l} + \mathbf{s}.$$

The parity of the state is determined by $(-)^N$ and the values of l are $l = N, N-2, \dots, 1$ or 0 depending on whether N is odd or even. The values of the spin quantum number are $\Sigma = \pm \frac{1}{2}$, and Λ is determined from the relation

$$\Omega = \Lambda + \Sigma. \quad (10)$$

The coefficients $A_{l\Omega\pm\frac{1}{2}}$, which are real, and the basic vectors $|Nl\Lambda\Sigma\rangle$, are given in the tables of SGN.

By decomposing the operators j_+ and j_- as $j_{\pm} = l_{\pm} + s_{\pm}$, the matrix elements occurring in (7) are obtained as below.

$$\left. \begin{aligned} \langle f | j_+ | i \rangle &= \sum_l \{ A_{l\Omega_f-\frac{1}{2}}^f A_{l\Omega_i-\frac{1}{2}}^i \sqrt{(l+\Omega_i+\frac{3}{2})(l-\Omega_i+\frac{1}{2})} + \\ &+ A_{l\Omega_f+\frac{1}{2}}^f A_{l\Omega_i+\frac{1}{2}}^i \sqrt{(l+\Omega_i+\frac{3}{2})(l-\Omega_i-\frac{1}{2})} + A_{l\Omega_f-\frac{1}{2}}^f A_{l\Omega_i+\frac{1}{2}}^i \} \quad \text{if } \Omega_f = \Omega_i + 1; \\ \langle f | j_- | i \rangle &= \sum_l \{ A_{l\Omega_f-\frac{1}{2}}^f A_{l\Omega_i-\frac{1}{2}}^i \sqrt{(l-\Omega_i+\frac{3}{2})(l+\Omega_i-\frac{1}{2})} + \quad \text{if } \Omega_f = \Omega_i - 1 \\ &+ A_{l\Omega_f+\frac{1}{2}}^f A_{l\Omega_i+\frac{1}{2}}^i \sqrt{(l+\Omega_i+\frac{1}{2})(l-\Omega_i+\frac{1}{2})} + A_{l\Omega_f+\frac{1}{2}}^f A_{l\Omega_i-\frac{1}{2}}^i \} \end{aligned} \right\} \quad (11)$$

In the case $\Omega = \frac{1}{2}$ the wave function (1) is

$$\Psi_{\frac{1}{2}} = \sqrt{\frac{2I+1}{16\pi^2}} \{ D_{M\frac{1}{2}}^I \chi_{\frac{1}{2}} + (-)^{I-J} D_{M-\frac{1}{2}}^I \chi_{-\frac{1}{2}} \} \quad (12)$$

and on account of the second term on the right side it is now possible for the operators j_+ and j_- to connect states (with the same parity) both of which have $K = \Omega = \frac{1}{2}$.

The matrix element

$$\langle \Omega_f = \frac{1}{2} | j_- I_+ + j_+ I_- | \Omega_i = \frac{1}{2} \rangle = (-)^{I-j} \frac{1}{2} (I + \frac{1}{2}) \{ \langle \chi_{-\frac{1}{2}}^f | j_- | \chi_{\frac{1}{2}}^i \rangle + \langle \chi_{\frac{1}{2}}^f | j_+ | \chi_{-\frac{1}{2}}^i \rangle \} \quad (13)$$

is computed by means of (9). The first and second parts of (13) are easily shown to be equal and the matrix element is given by

$$\begin{aligned} \langle \Omega_f = \frac{1}{2} | j_- I_+ + j_+ I_- | \Omega_i = \frac{1}{2} \rangle &= \\ &= (-)^{I-j} (I + \frac{1}{2}) \sum_l \{ A_{l0}^i A_{l0}^f + \sqrt{l(l+1)} (A_{l0}^i A_{l1}^f + A_{l1}^i A_{l0}^f) \}. \end{aligned} \quad (14)$$

Besides this coupling of H_{pert} , the usual coupling of j_+ to intrinsic states with $\Omega_f = \frac{3}{2}$ is obtained from formula (11).

The coefficients $A_{l\Omega \pm \frac{1}{2}}$ and the energies E_i and E_f are given in SGN only for the values $\pm 2, \pm 4, \pm 6$ of the deformation parameter. Interpolations are thus necessary to get the values of the denominator and numerator in the expression for δJ^{orb} , if the method above is used.

A better understanding of the empirically encountered approximate selection rules in using these tabulated single particle states can be reached by the use of what is called the asymptotic representation. The use of this representation facilitates the calculations and gives the matrix elements and energies in a closed form for every value of the deformation parameter.

Asymptotic solutions for strongly deformed nuclei

In the reference SGN an asymptotic solution of the wave equation is studied in the limit of very strong deformations. It is then suitable to treat as perturbations the $\mathbf{l}^2$ and $\mathbf{l} \cdot \mathbf{s}$ terms of the single-particle Hamiltonian, employed in this reference.

The total Hamiltonian is written

$$\left. \begin{aligned} H &= H_0 - \kappa \hbar \hat{\omega}_0 (2\mathbf{l} \cdot \mathbf{s} + \mu \mathbf{l}^2) \\ H_0 &= -\frac{\hbar^2}{2M} \Delta + \frac{M}{2} (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \\ H_0 &= H_x + H_y + H_z \end{aligned} \right\} \quad (15)$$

The choice of the parameters κ and μ is discussed in SGN. The frequencies are expressed in terms of the deformation parameter ε through

$$\begin{aligned} \omega_x = \omega_y = \omega_0(\varepsilon) (1 + \frac{1}{3} \varepsilon) \\ \omega_z = \omega_0(\varepsilon) (1 - \frac{2}{3} \varepsilon) \end{aligned} \quad \hat{\omega}_0 = \omega_0(\varepsilon = 0).$$

The parameter ε has the relation $\varepsilon \approx 0.95 \beta$ to the parameter β used by A. Bohr and B. R. Mottelsson. The condition of constant volume of the nucleus leads to $\omega_x \omega_y \omega_z = \text{const}$ or

$$\omega_0(\varepsilon) \approx \omega_0 (1 + \frac{1}{9} \varepsilon^2). \quad (16)$$

The energies of the Hamiltonian H_0 are given by

$$E_{n_z n_1} = \hbar \omega_0 (\varepsilon) \left[\left(N + \frac{3}{2} \right) + \varepsilon \frac{n_1 - 2n_z}{3} \right] \quad (17)$$

where $N = n_x + n_y + n_z$ is the total oscillator quantum number and $n_1 = n_x + n_y$. The product vectors $|n_x\rangle |n_y\rangle |n_z\rangle$ form a representation with H_0 diagonal. Because of the symmetry around the z -axis, the energies are degenerate of the order $n_1 + 1$.

It is useful to introduce as a basic set linear combinations of the normalized $|n_x\rangle$ and $|n_y\rangle$ vectors

$$|n_1 \Lambda\rangle = \sum_{n_x, n_y} a_{n_x n_y} |n_x\rangle |n_y\rangle \quad (n_x + n_y = n_1), \quad (18)$$

where the combinations (cf. appendix B of SGN) are chosen in such a way that

$$l_z |n_1 \Lambda\rangle = \Lambda |n_1 \Lambda\rangle.$$

Each single particle level is now designated by $n_z, n_1, \Lambda, \Sigma$ with $\Omega = \Lambda + \Sigma$. The values of Λ are thus equal to $\Omega \pm \frac{1}{2}$ and are found to be even or odd as n_1 is even or odd. Λ and n_z are thus considered as "good" quantum numbers in the limit of strong deformations.

The operators $j_{\pm} = l_{\pm} + s_{\pm}$ give rise to couplings through which Λ and Σ are changed by one unit. The corresponding matrix elements $\langle f | j_{\pm} | i \rangle$ can be obtained by starting from the differential equation for the harmonic oscillator in cylindrical coordinates (Pauling and Wilson, Introduction to Quantum Mechanics, p. 111 (1935)). The differential equation for the radial part of the wave function given there can be transformed to the differential equation for associated Laguerre polynomial of degree $r - \Lambda$, where r is given by $r = \frac{n_1 + \Lambda}{2}$.

The base vectors are written

$$|n_z n_1 \Lambda \Sigma; \rho, z, \varphi\rangle = |r \Lambda; \rho\rangle |n_z; z\rangle |\Lambda; \varphi\rangle$$

$$\text{with } |\Lambda; \varphi\rangle = \frac{1}{\sqrt{2\pi}} e^{i\Lambda\varphi} \quad \text{and} \quad |n_z; z\rangle = \left(\frac{1}{\pi^{\frac{1}{2}} 2^{n_z} n_z!} \right)^{\frac{1}{2}} e^{-\frac{M\omega_z}{2\hbar} z^2} H_{n_z} \left(\sqrt{\frac{M\omega_z}{\hbar}} z \right).$$

By using the generating function for the Laguerre polynomials it is possible to determine all matrix elements $\langle r' \Lambda' | \eta^k | r \Lambda \rangle$ ($\eta = \rho^2$ and k half integer) from the formula

$$\begin{aligned} & \sum_{r=\Lambda}^{\infty} \sum_{r'=\Lambda'}^{\infty} \frac{(z_1)^r (z_2)^{r'}}{(r)! (r')!} \frac{\langle r' \Lambda' | \eta^k | r \Lambda \rangle}{N_{r\Lambda} \cdot N_{r'\Lambda'}} = \\ & = \frac{1}{2} (-1)^{\Lambda+\Lambda'} (1-z_1)^{k+\frac{\Lambda'-\Lambda}{2}} (1-z_2)^{k+\frac{\Lambda-\Lambda'}{2}} \sum_{s=0}^{\infty} (z_1)^{s+\Lambda} (z_2)^{s+\Lambda'} \frac{\left(\frac{\Lambda+\Lambda'}{2} + k + s \right)!}{s!} \end{aligned}$$

where generating functions in terms of z_1 and z_2 have been employed. The normalization factors for the radial wave functions are given by

$$N_{r\Lambda} = \frac{\sqrt{2(r-\Lambda)!}}{\sqrt{(r!)^3}}$$

The matrix elements $\langle r' \Lambda' | d/d\eta | r \Lambda \rangle$ are obtained by using the well-known relations involving Laguerre polynomials.

The operators l_+ and l_- are written in cylindrical coordinates as follows

$$l_{\pm} = e^{\pm i\varphi} \left[-\rho \frac{\partial}{\partial z} + \frac{\partial}{\partial \rho} \pm \frac{i}{\rho} \frac{\partial}{\partial \varphi} \right]$$

and the matrix elements, needed in the formula for δJ , are found to be¹

$$\left. \begin{aligned} \langle n_z - 1 \ n_1 + 1 \ \Lambda + 1 \ \Sigma | l_+ | n_z \ n_1 \ \Lambda \ \Sigma \rangle &= \sqrt{n_z (n_1 + \Lambda + 2)}, \\ \langle n_z + 1 \ n_1 - 1 \ \Lambda + 1 \ \Sigma | l_+ | n_z \ n_1 \ \Lambda \ \Sigma \rangle &= \sqrt{(n_z + 1) (n_1 - \Lambda)}, \\ \langle n_z - 1 \ n_1 + 1 \ \Lambda - 1 \ \Sigma | l_- | n_z \ n_1 \ \Lambda \ \Sigma \rangle &= \sqrt{n_z (n_1 - \Lambda + 2)}, \\ \langle n_z + 1 \ n_1 - 1 \ \Lambda - 1 \ \Sigma | l_- | n_z \ n_1 \ \Lambda \ \Sigma \rangle &= \sqrt{(n_z + 1) (n_1 + \Lambda)}, \\ \langle n_z \ n_1 \ \Lambda \ \Sigma \pm 1 | s_{\pm} | n_z \ n_1 \ \Lambda \ \Sigma \rangle &= 1. \end{aligned} \right\} \quad (19)$$

There are thus in the asymptotic case at most five couplings of the operators j_+ and j_- . The energies E_i and E_f of the single-particle states are, to first order determined by (17) and by the diagonal matrix elements of the perturbation operators $\mathbf{l} \cdot \mathbf{s}$ and $\mathbf{l}^2$.

$$\left. \begin{aligned} \langle n_z \ n_1 \ \Lambda \ \Sigma | \mathbf{l} \cdot \mathbf{s} | n_z \ n_1 \ \Lambda \ \Sigma \rangle &= \Lambda \Sigma, \\ \langle n_z \ n_1 \ \Lambda \ \Sigma | \mathbf{l}^2 | n_z \ n_1 \ \Lambda \ \Sigma \rangle &= \Lambda^2 + 2 n_1 n_z + 2 n_z + n_1. \end{aligned} \right\} \quad (20)$$

By writing these operators in the form

$$\begin{aligned} \mathbf{l} \cdot \mathbf{s} &= l_z s_z + \frac{1}{2} (l_+ s_- + l_- s_+), \\ \mathbf{l}^2 &= l_z^2 + \frac{1}{2} (l_+ l_- + l_- l_+) \end{aligned}$$

the non-diagonal elements are then easily obtained by employing eqs. (19)

$$\left. \begin{aligned} \langle n_z - 2 \ n_1 + 2 \ \Lambda \ \Sigma | \mathbf{l}^2 | n_z \ n_1 \ \Lambda \ \Sigma \rangle &= \sqrt{n_z (n_z - 1) (n_1 + \Lambda + 2) (n_1 - \Lambda + 1)}, \\ \langle n_z + 2 \ n_1 - 2 \ \Lambda \ \Sigma | \mathbf{l}^2 | n_z \ n_1 \ \Lambda \ \Sigma \rangle &= \sqrt{(n_z + 2) (n_z + 1) (n_1 + \Lambda) (n_1 - \Lambda)}, \\ \langle n_z - 1 \ n_1 + 1 \ \Lambda + 1 \ \Sigma \mp 1 | \mathbf{l} \cdot \mathbf{s} | n_z \ n_1 \ \Lambda \ \Sigma \rangle &= \frac{1}{2} \sqrt{n_z (n_1 \pm \Lambda + 2)}, \\ \langle n_z + 1 \ n_1 - 1 \ \Lambda - 1 \ \Sigma \mp 1 | \mathbf{l} \cdot \mathbf{s} | n_z \ n_1 \ \Lambda \ \Sigma \rangle &= \frac{1}{2} \sqrt{(n_z + 1) (n_1 \mp \Lambda)}. \end{aligned} \right\} \quad (21)$$

The second-order contribution to the energy is given by

$$\Delta E_i^{\text{II}} = \kappa^2 (\hbar \omega_0)^2 \left\{ \sum_f \left(\frac{4 |\langle f | \mathbf{l} \cdot \mathbf{s} | i \rangle|^2}{E_i - E_f} + \mu^2 \frac{|\langle f | \mathbf{l}^2 | i \rangle|^2}{E_i - E_f} \right) \right\}. \quad (22)$$

In calculating the energy values (22) the contribution from the diagonal parts of the operators $\mathbf{l} \cdot \mathbf{s}$ and $\mathbf{l}^2$ are suitably included in the denominators, as the lowest order energy values of eq. (17) are not sufficiently accurate.

¹ These matrix elements have recently also been obtained by A. J. Rassey, Phys. Rev., 109, (1958).

A calculation of the first- and second-order contribution to the energy has been performed for the case $N=6$, $\mu=0.45$, $\eta=6$ and the result is exhibited in fig. F. These energies are compared in the figure with those obtained by an exact diagonalisation (levels to the right). The close agreement is remarkable.

Comparison between theory and experiments

By means of the formulas given above, it is now possible to calculate the single-particle contribution δJ^{orb} to the moment of inertia. This has been done for nuclei in the mass regions $A=25$, $153 \leq A \leq 187$ and $229 \leq A \leq 245$. The results are found in the tables I and II (last two columns). The whole difference in moments of inertia between an odd- A nucleus and the preceding even-even nucleus is considered as a sum of the orbital contribution of the single particle and the contribution of the core, due to the change of shape induced by the single particle.

$$\delta J = \delta J^{\text{orb}} + \delta J^{\text{coll}}. \quad (23)$$

This is to be compared with the experimentally found difference

$$\delta J^{\text{exp}} = J^{\text{odd}} - J^{\text{e-e}}. \quad (24)$$

The values of $(3\hbar^2/J)_{\text{e-e}}$ and $(3\hbar^2/J)_{\text{odd}}$ are obtained from data on rotational energies. The contributions δJ^{exp} and δJ^{orb} are expressed in units of the value J_{rig} , which is the moment of inertia associated with a rigid rotation of an ellipsoid of constant density and of mass M about an axis perpendicular to the symmetry axis (A. Bohr and B. R. Mottelson, Dan. Mat. Fys. Medd., 30, no. 1, 1955).

$$\left. \begin{aligned} J_{\text{rig}} &= \frac{2}{5} M A R_0^2 (1 + 0.33 \varepsilon), \\ R_0 &= 1.2 \cdot 10^{-13} A^{\frac{1}{3}} \text{ (cm)}. \end{aligned} \right\} \quad (25)$$

The values of the quantity $\hbar \dot{\omega}_0$ for different nuclei are given by $\hbar \dot{\omega}_0 \sim 41 A^{-\frac{1}{3}}$ (MeV). The nuclear deformation parameter ε has been obtained (for different mass number) from B. R. Mottelson and S. G. Nilsson, Dan. Mat. Fys. Medd. (forthcoming) and the parameter η is determined from

$$\eta = \frac{\varepsilon}{\varkappa} \cdot \frac{\omega_0(\varepsilon)}{\omega_0} \approx \frac{\varepsilon}{\varkappa} (1 + \frac{1}{3} \varepsilon^2). \quad (26)$$

The parameters $\varkappa$ and μ , which characterize the single-particle potential, are those given by S. G. N.

<i>Odd neutron nucleus</i>					
Neutron number	$\varkappa$	Shell	μ	Shell	μ
$82 \leq N \leq 126$	0.05	$N=5$	0.45	$N=6$	0.45
$N > 126$	0.05	$N=6$	0.45	$N=7$	0.40
<i>Odd proton nucleus</i>					
Proton number	$\varkappa$	Shell	μ	Shell	μ
$Z=13$	0.08	$N=2$	0.00	$N=3$	0.00
$50 \leq Z \leq 82$	0.061	$N=4$	0.55	$N=5$	0.45
$82 \leq Z \leq 126$	0.05	$N=5$	0.70	$N=6$	0.45

Discussion

As pointed out above, in addition to the orbital contribution of the last odd nucleon to the moment of inertia, there is also a collective contribution due to the change in shape of the nucleus caused by the addition of the last nucleon. This latter contribution is, according to the calculations, in general rather small except in a few cases. In most of these exceptional cases one usually also expects a large δJ^{coll} on the basis of the assigned last particle orbitals. One expects an orbital to have a polarizing effect on the core if the corresponding energy level is a steep function of the deformation (see figs. 3-6 in B. R. Mottelsson and S. G. Nilsson, *Dan. Mat. Fys. Medd.*, forthcoming).

One approximate way of correcting for δJ^{coll} is to use an effective value of J_{e-e} , obtained by interpolating between the values of the moment of inertia associated with even-even nuclei with mass values just above and just below the odd nucleus (i.e. $A-1$ and $A+1$). In fig. A the experimental quantities $(3\hbar^2/J)_{e-e}$ for even-even nuclei in the region $90 \leq N \leq 112$ are plotted with Z as a parameter. It is seen from this figure that the moment of inertia has a strong dependence on the neutron number for $Z=62, 64, 66$ and 74 and on the proton number for $N=96, 98, 104$ and 106 . Here an interpolation between the values of $(3\hbar^2/J)_{e-e}$ for the preceding and the following even-even nucleus gives in most cases a corrected δJ^{exp} , which is then in better agreement with δJ^{orb} . This is for instance the case for ${}_{64}\text{Gd}^{155}$, ${}_{64}\text{Gd}^{157}$, ${}_{66}\text{Dy}^{161}$, ${}_{67}\text{Ho}^{165}$, ${}_{71}\text{Lu}^{175}$ and ${}_{73}\text{Ta}^{181}$.

For some nuclei where also a large discrepancy between δJ^{exp} and δJ^{orb} is found, the situation is a little more complicated. An example is Ho^{165} , which according to the model has an odd proton in the $[523\ 7/2]$ orbital. A study of the nuclei adjacent to ${}_{67}\text{Ho}^{165}$, namely ${}_{66}\text{Dy}^{164}$ and ${}_{68}\text{Er}^{166}$, shows that $3\hbar^2/J$ varies from 73 to 81 and this variation does not appear to be explained on the basis of the polarization of the proton orbital corresponding to 67 protons as the corresponding energy level does not vary with the deformation. On the other hand, the refilling of the broken $h_{11/2}$ shell is expected to occur at about the neutron number 98. A calculation of the total energy of the nucleus on the basis of calculated single-particle energies shows in this case two competing minima corresponding to different deformations, one with the $11/2$ state filled, the other with the same state unfilled. One or two additional protons may change the field in a way that disturbs this sensitive competition and may thus give a large contribution to the collective moment of inertia.

Another question which must be taken into account is whether the perturbation treatment is sufficiently accurate for all couplings involved. In the derivation of formula (7), the rotational energies were neglected in the denominators compared with the single-particle energies. This approximation is valid if the energies of the initial and final configurations are very different. But if there are close lying configurations, which are coupled by the operators j_+ and j_- , the perturbation treatment developed above is not suitable.

Such a situation of two close-lying coupled energy states is treated in G. Herzberg, *Spectra of Diatomic Molecules* p. 283 (1950). A. K. Kerman (*Dan. Mat. Fys. Medd.*, 30, no. 15 (1956)) has used the diagonalization method of this reference to determine the coupling between the $1/2$ - and $3/2$ - rotational bands of the nucleus W^{183} . The corresponding single-particle orbitals of ref. SGN are

[510 1/2] and [512 3/2]. The spectra have been measured very accurately and the effective moments of inertia for the two bands are given by

$$\frac{3\hbar^2}{J_{1/2}} = 78.2 \text{ (keV)}, \quad \frac{3\hbar^2}{J_{3/2}} = 99.5 \text{ (keV)}.$$

This is to be compared with the even-even nucleus W^{182} , where $3\hbar^2/J = 100$ (keV). The calculations by Kerman give the following values, which characterize the two bands when their mutual coupling is subtracted

$$\frac{3\hbar^2}{J_{1/2}} = 95.1 \text{ (keV)}, \quad \frac{3\hbar^2}{J_{3/2}} = 84.3 \text{ (keV)}.$$

These corrected empirical values and the theoretical values of δJ^{orb} , in the absence of the coupling between these nearlylying states, are given in parenthesis in table I and have been plotted in fig. B.

A study of the diagrams of energy levels of Mottelson–Nilsson figs. 2–6 (of cited work in press) reveals many other such pairs of close-lying orbitals, coupled by the Coriolis forces.

Examples of such pairs are, e.g. [211 1/2] and [200 1/2], [200 1/2] and [211 3/2], [413 5/2] and [411 3/2], [404 7/2] and [402 5/2], [521 3/2] and [523 5/2], [743 7/2] and [752 5/2], [622 5/2] and [624 7/2], [633 5/2] and [631 3/2]. Mixed rotational spectra have also been observed in many of the corresponding nuclei. A preliminary investigation indicates that if these special couplings are treated according to the method above, a better agreement between the values of δJ^{exp} and δJ^{orb} will be obtained in most cases.

It may also be noted in this connection that a negative contribution to δJ results from the coupling of a higher-lying configuration to an unoccupied lower one. This is illustrated in the case of W^{183} above.

The author wants to express his thanks to Dr. B. R. Mottelson for suggesting this investigation and to him and Dr. S. G. Nilsson for giving access to an article about the classification of the spectra of deformed nuclei in advance of publication. The author is also very thankful to Prof. T. Gustafson for his continued interest in this work and to Dr. S. G. Nilsson for many helpful discussions, suggestions and comments.

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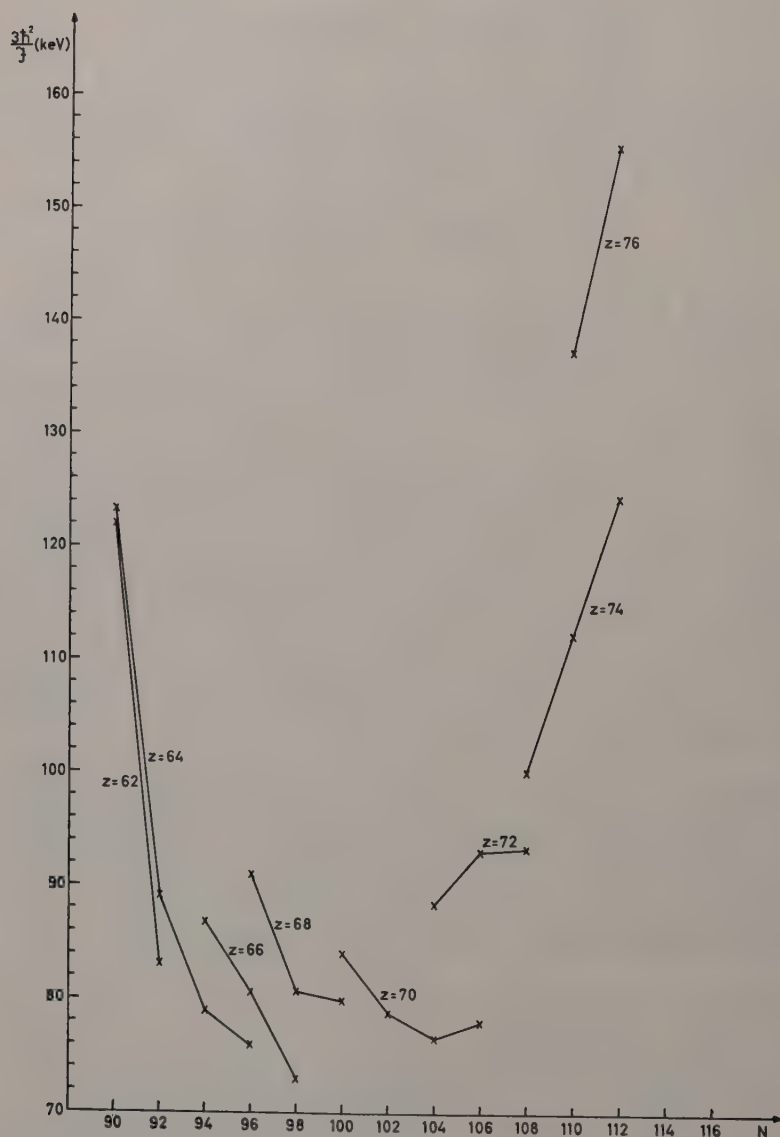


Fig. A. Energies of first excited rotational states ($= 3\hbar^2/J$) in even-even nuclei, as a function of N ($90 \leq N \leq 112$) with Z as a parameter.

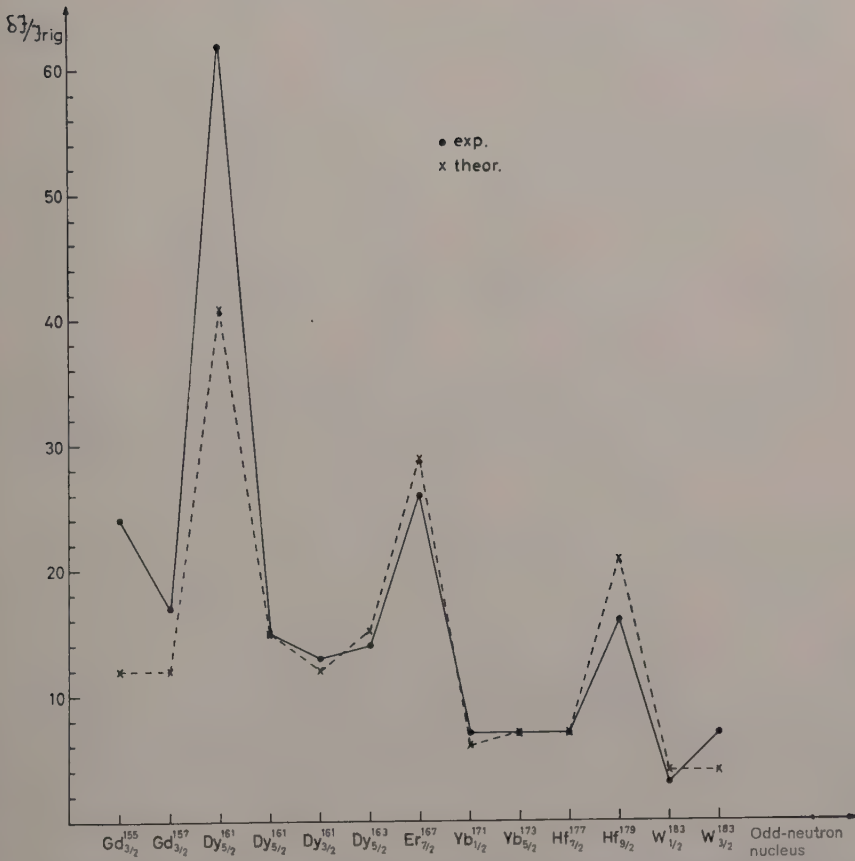


Fig. B. The values of $\delta J/J_{rig}$ are given in per cent. The first value of Dy^{161} refers to the state $[642 \frac{5}{2}^+]$, the second one to the state $[523 \frac{5}{2}^-]$.

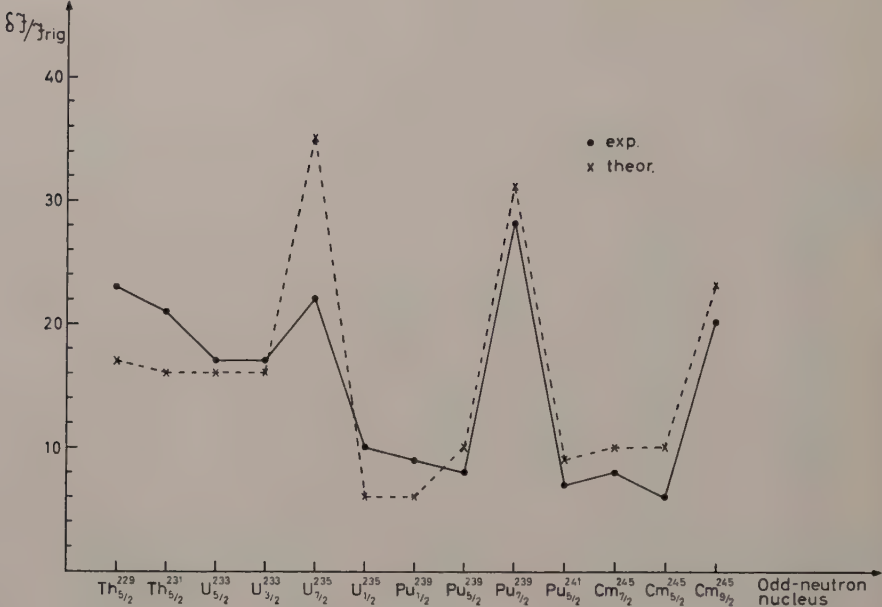


Fig. C.

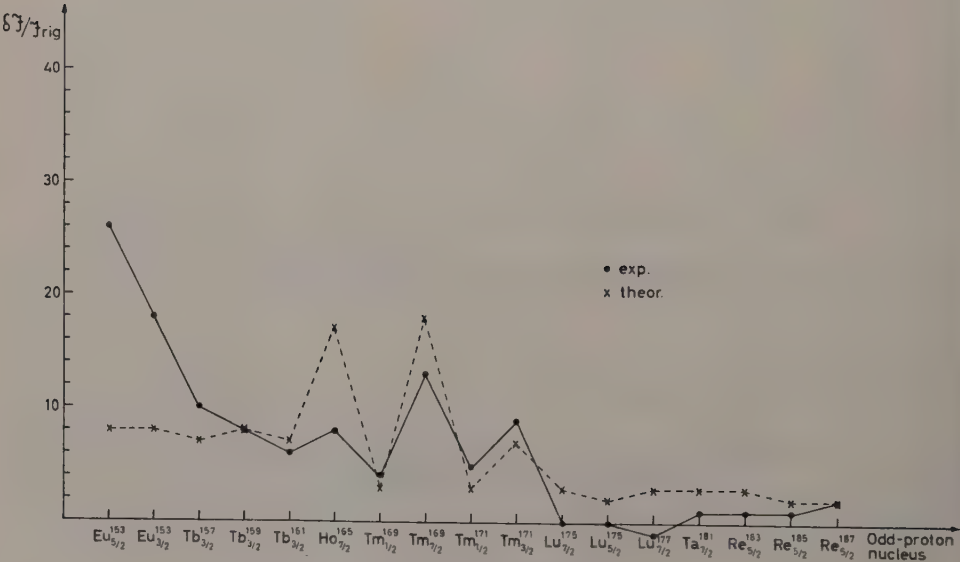


Fig. D.

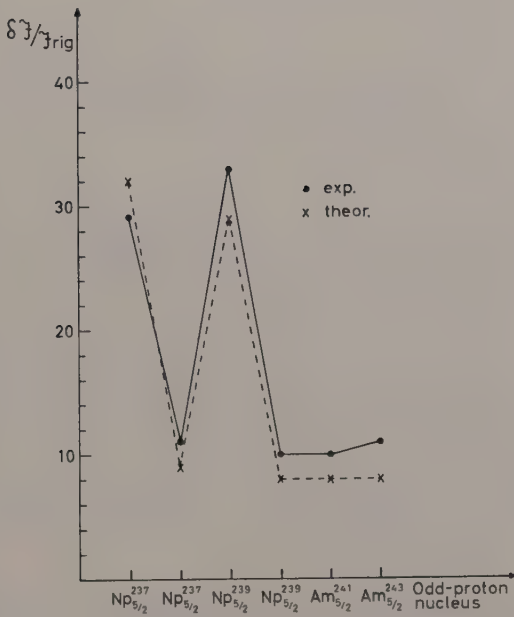


Fig. E. The first values of Np^{237} and Np^{239} refer to the state $[642 \frac{5}{2} +]$, the second ones to the state $[523 \frac{5}{2} -]$.

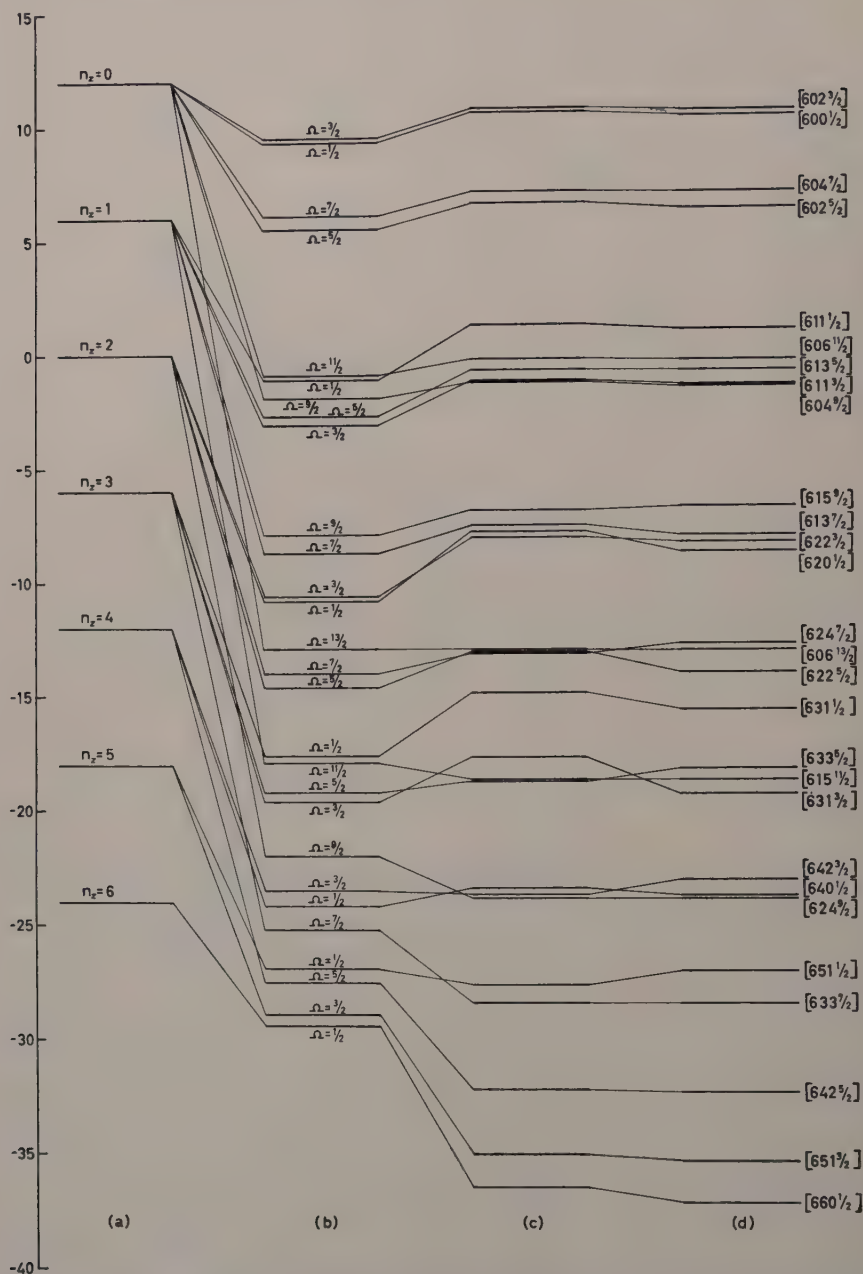


Fig. F. Energy levels (in units of $\kappa \hbar \omega_0$) of the single-particle states belonging to the Shell $N = 6$.
 (a) Levels obtained from eq. (17).
 (b) Levels obtained by including the first-order contribution (formula (20)).
 (c) Levels obtained by including the second-order contribution (formula (22)).
 (d) Levels obtained from an exact diagonalisation (cf. ref. SGN).

Captions to tables I and II

Column 2: The single particle orbitals are designated by the quantum numbers $[N n_z \Lambda \Omega]$ and this assignment has been obtained from the article in Dan. Mat Fys. Medd. by B. R. Mottelson and S. G. Nilsson (forthcoming).

Columns 3, 9: The empirical evidence has been taken from the following references:

- a. B. R. Mottelson and S. G. Nilsson, Dan. Mat. Fys. Medd. (forthcoming).
- b. Alder, Bohr, Huus, Mottelson and Winther, Rev. of Mod. Phys., Vol. 28, No. 4, 530 (1956).
- c. Mihelich, Harmatz and Handley, Phys. Rev., 108, 998 (1957).
- d. Chupp, Du Mond, Gordon, Jopson and Mark, Bull. Am. Phys. Soc., II: 3, 55 (1958).
- e. J. M. Hollander, Phys. Rev., 103, 1591 (1956).
- f. Smith, Gibson, Hollander, Phys. Rev., 105, 1517 (1957).
- g. E. N. Hatch and F. Boehm, Phys. Rev., 108, 114 (1957).
- h. O. B. Nielsen *et al.* (1958), to be published.
- i. I. Perlman and J. O. Rasmussen, Chapter on Alpha Radioactivity in Handbuch der Physik. (1957). Vol. 42 (Springer Verlag, Berlin).
- j. P. M. Endt and C. M. Braams, Rev. of Mod. Phys., Vol. 29, No. 4, 683 (1957).
- k. Strominger, Hollander, Seaborg, Table of isotopes, Rev. of Mod. Phys. (1958), in press.

Columns 4, 7, 9: The rotational parameters have been calculated according to the formulas (4) and (8).

Columns 5, 6: The assumed values of the deformation parameter ε have been taken from ref. (a) above and the parameter η is determined from the relation (26).

Column 9: This column lists the moments of inertia of the even-even nuclei, obtained by removing the last odd particle.

Columns 11, 12, 13: The values of δJ are expressed in per cent of J_{rig} .

Column 12: In calculating δJ^{orb} , formulas (11) and (14) have been used. Interpolations are necessary, as pointed out in the text.

Column 13: In calculating the values exhibited in the last column, denoted $\delta J_{\text{as}}^{\text{orb}}$, the approximate "asymptotic" expressions have been used. The calculations of the energy differences $(E_f - E_i)$ have been performed to first order, except for the particle states [743 7/2] and [642 5/2]. In these cases the second-order contributions to the energy differences are great and have been included.

Table I: Contribution to the moment of inertia from the last odd particle. Odd-*N* nuclei

Nucleus	Assigned orbital	Levels in rotational band (keV)	a_{exp}	ε	η	$\left(\frac{3\hbar^2}{J}\right)_{\text{odd}}$ (keV)	Ref.	$\left(\frac{3\hbar^2}{J}\right)_{\text{e-e}}$ (keV)	Ref.	$\frac{\delta J^{\text{exp}}}{J_{\text{rig}}}$	$\frac{\delta J^{\text{orb}}}{J_{\text{rig}}}$	δ
$^{91}_{64}\text{Gd}^{155}$	[521 3/2]	7/2 — 146 5/2 — 60 3/2 — 0		0.31	6.3	72.0	a	123.2	c	26	12	
$^{93}_{64}\text{Gd}^{157}$	[521 3/2]	7/2 — 131 5/2 — 55 3/2 — 0		0.31	6.3	66.0	a	89.1	c	17	12	
$^{95}_{66}\text{Dy}^{161}$	[642 5/2]	9/2 — 102 7/2 — 44 5/2 — 0		0.30	6.1	37.7	a	87.0	c	62	41	
$^{95}_{66}\text{Dy}^{161}$	[523 5/2]	9/2 — 203 7/2 — 103 5/2 — 26		0.30	6.1	66.0	a	87.0	c	15	15	
$^{95}_{66}\text{Dy}^{161}$	[521 3/2]	5/2 — 132 3/2 — 75		0.30	6.1	68.4	a	87.0	c	13	12	
$^{97}_{68}\text{Dy}^{163}$	[523 5/2]	9/2 — 167 7/2 — 74 5/2 — 0		0.30	6.1	63.4	a	80.8	c	14	15	
$^{99}_{68}\text{Er}^{167}$	[633 7/2]	11/2 — 172 9/2 — 78 7/2 — 0		0.29	5.9	52.0	a	80.7	c	27	29	
$^{101}_{70}\text{Yb}^{171}$	[521 1/2]	5/2 — 76 3/2 — 67 1/2 — 0	0.87	0.28	5.7	72.4	a	84.2	c	7	6	
$^{103}_{70}\text{Yb}^{173}$	[512 5/2]	9/2 — 180 7/2 — 79 5/2 — 0		0.28	5.7	67.7	a	78.7	c	7	7	
$^{105}_{72}\text{Hf}^{177}$	[514 7/2]	11/2 — 250 9/2 — 113 7/2 — 0		0.27	5.5	75.3	a	88.3	d	7	7	
$^{107}_{72}\text{Hf}^{179}$	[624 9/2]	13/2 — 262 11/2 — 121 9/2 — 0		0.26	5.3	66.0	a	93.1	d	16	21	
$^{109}_{74}\text{W}^{183}$	[510 1/2]	7/2 — 207 5/2 — 99.1 3/2 — 46.5 1/2 — 0	0.19	0.21	4.2	78.1 (95.1)	b	100	b	10 (3)	7 (4)	
$^{109}_{74}\text{W}^{183}$	[512 3/2]	7/2 — 412.1 5/2 — 291.7 3/2 — 208.8		0.21	4.2	99.5 (84.3)	b	100	b	0 (7)	8 (4)	
$^{139}_{90}\text{Th}^{229}$	[633 5/2]	9/2 — 99 7/2 — 43 5/2 — 0		0.22	4.5	36.9	i	58	h	23	17	
$^{141}_{90}\text{Th}^{231}$	[633 5/2]	7/2 — 42 2/2 — 0		0.23	4.6	36.0	i	53	k	21	16	

e I (continued)

Isotope	Assigned orbital	Levels in rotational band (keV)	a_{exp}	ε	η	$\left(\frac{3\hbar^2}{J}\right)_{\text{odd}}$ (keV)	Ref.	$\left(\frac{3\hbar^2}{J}\right)_{\text{e-e}}$ (keV)	Ref.	$\frac{\delta J^{\text{exp}}}{J_{\text{rig}}}$	$\frac{\delta J^{\text{orb}}}{J_{\text{rig}}}$	$\frac{\delta J_{\text{as}}^{\text{orb}}}{J_{\text{rig}}}$
U^{233}	[633 5/2]	9/2 — 92 7/2 — 40 5/2 — 0		0.24	4.7	34.3	a	47.2	e	17	16	17
U^{233}	[631 3/2]	5/2 — 342 3/2 — 313		0.24	4.7	34.8	a	47.2	e	17	16	18
U^{235}	[743 7/2]	11/2 — 103 9/2 — 46 7/2 — 0		0.24	4.9	30.7	a	43.7	e	22	35	24
U^{235}	[631 1/2]	7/2 — 84 5/2 — 52 3/2 — 14 1/2 — 0.08	-0.26	0.24	4.9	36.7	a	43.7	f	10	6	6
Pu^{239}	[631 1/2]	7/2 — 76 5/2 — 57 3/2 — 8 1/2 — 0	-0.58	0.26	5.3	37.4	a	44.2	f	9	6	6
Pu^{239}	[622 5/2]	9/2 — 388 7/2 — 331 5/2 — 286		0.26	5.3	38.6	a	44.2	f	8	10	9
Pu^{239}	[743 7/2]	11/2 — 483 9/2 — 434 7/2 — 392		0.26	5.3	28.0	a	44.2	f	28	31	21
Pu^{241}	[622 5/2]	7/2 — 44 5/2 — 0		0.27	5.4	37.7	a	42.9	f	7	9	9
Cm^{245}	[624 7/2]	11/2 — 124 9/2 — 55 7/2 — 0		0.26	5.3	36.7	a	42.6	e	8	10	10
Cm^{245}	[622 5/2]	9/2 — 357 7/2 — 301 5/2 — 257		0.26	5.3	37.7	a	42.6	e	6	10	9
Cm^{245}	[734 9/2]	13/2 — 511 11/2 — 450 9/2 — 394		0.26	5.3	30.5	a	42.6	e	19	23	17

Table II: Contribution to the moment of inertia from the last odd particle. Odd-Z nu

Nucleus	Assigned orbital	Levels in rotational band (keV)	a_{exp}	ε	η	$\left(\frac{3\hbar^2}{J}\right)_{\text{odd}}$ (keV)	Ref.	$\left(\frac{3\hbar^2}{J}\right)_{\text{e-e}}$ (keV)	Ref.	$\frac{\delta J^{\text{exp}}}{J_{\text{rig}}}$	$\frac{\delta J^{\text{orb}}}{J_{\text{rig}}}$	$\frac{\delta J^{\text{vib}}}{J_{\text{rig}}}$
$^{90}_{63}\text{Eu}^{153}$	[413 5/2]	9/2 — 190 7/2 — 83 5/2 — 0		0.30	5.0	71.1	a	122	b	26	8	
$^{90}_{63}\text{Eu}^{153}$	[411 3/2]	5/2 — 172 3/2 — 103		0.30	5.0	82.8	a	122	b	18	8	
$^{92}_{65}\text{Tb}^{157}$	[411 3/2]	7/2 — 144 5/2 — 61 3/2 — 0		0.31	5.2	73.2	a	89	b	10	7	
$^{94}_{65}\text{Tb}^{159}$	[411 3/2]	7/2 — 138 5/2 — 58 3/2 — 0		0.31	5.2	69.6	a	79	b	8	8	
$^{96}_{65}\text{Tb}^{161}$	[411 3/2]	5/2 — 57 3/2 — 0		0.31	5.2	68.4	a	76	b	6	7	
$^{98}_{67}\text{Ho}^{165}$	[523 7/2]	11/2 — 212 9/2 — 95 7/2 — 0		0.30	5.0	63.3	a	73	b	8	17	
$^{100}_{69}\text{Tm}^{169}$	[411 1/2]	7/2 — 139.0 5/2 — 118.2 3/2 — 8.4 1/2 — 0	-0.77	0.28	4.7	74.3	g	80	b	4	3	
$^{100}_{69}\text{Tm}^{169}$	[523 7/2]	9/2 — 473 7/2 — 379		0.28	4.7	62.7	a	80	b	13	18	
$^{102}_{69}\text{Tm}^{171}$	[411 1/2]	7/2 — 129.1 5/2 — 116.7 3/2 — 5.1 1/2 — 0	-0.86	0.28	4.7	72.1	g	80	b	5	3	
$^{102}_{69}\text{Tm}^{171}$	[411 3/2]	5/2 — 744 3/2 — 688		0.28	4.7	67.2	a	80	b	9	7	
$^{104}_{71}\text{Lu}^{175}$	[404 7/2]	11/2 — 251 9/2 — 114 7/2 — 0		0.28	4.7	76.0	a	76.6	c	0	3	
$^{104}_{71}\text{Lu}^{175}$	[402 5/2]	7/2 — 433 5/2 — 343		0.28	4.7	77.1	a	76.6	c	0	2	
$^{106}_{71}\text{Lu}^{177}$	[404 7/2]	9/2 — 119 7/2 — 0		0.26	4.3	79.3	a	78	b	—1	3	
$^{108}_{73}\text{Ta}^{181}$	[404 7/2]	9/2 — 136 7/2 — 0		0.23	3.8	90.7	a	93	b	1	3	
$^{108}_{75}\text{Re}^{183}$	[402 5/2]	9/2 — 260 7/2 — 114 5/2 — 0		0.21	3.5	97.7	a	100	b	1	3	
$^{108}_{75}\text{Re}^{183}$	[514 9/2]	11/2 — 664 9/2 — 496		0.21	3.5	91.6	a	100	b	3*	13*	

II (continued)

Isotop	Assigned orbital	Levels in rotational band (keV)	a_{exp}	ε	η	$\left(\frac{3\hbar^2}{J}\right)_{\text{odd}}$ (keV)	Ref.	$\left(\frac{3\hbar^2}{J}\right)_{\text{e-e}}$ (keV)	Ref.	$\frac{\delta J^{\text{exp}}}{J_{\text{rig}}}$	$\frac{\delta J^{\text{orb}}}{J_{\text{rig}}}$	$\frac{\delta J^{\text{orb}}}{J_{\text{rig}}}$
Ce^{185}	[402 5/2]	9/2 — 286 7/2 — 125 5/2 — 0		0.19	3.1	107.1	a	112	b	1	2	2
Ce^{187}	[402 5/2]	9/2 — 300 7/2 — 134 5/2 — 0		0.19	3.1	114.9	d	124	b	2	2	2
Al^{25}	[202 5/2]	9/2 — 3440 7/2 — 1610 5/2 — 0		0.35	4.5	1380	a	1368	j	0	15	—
Al^{25}	[211 1/2]	5/2 — 1810 3/2 — 950 1/2 — 450		0.41	5.3	1016	a	1368	j	23	24	—
Al^{25}	[200 1/2]	5/2 — 3880 3/2 — 2700 1/2 — 2500	-0.02	0.37	4.7	908	a	1368	j	33	17	—
Al^{25}	[330 1/2]	1/2 — 3850 7/2 — 3720 3/2 — 3090	-0.56	0.55	7.2	682	a	1368	j	63	39	—
Al^{25}	[211 3/2]	5/2 — 4600 3/2 — 4220	-3.2	0.39	5.0	453	a	1368	j	131	47	—
Pa^{233}	[530 1/2]	5/2 — 69 7/2 — 56 1/2 — 7 3/2 — 0	-1.31	0.24	4.8	35.8	a	52.0	b	20*	5*	6*
Pa^{233}	[642 5/2]	7/2 — 103 5/2 — 86		0.24	4.8	14.6	a	52.0	b	112*	35*	20*
Np^{237}	[642 5/2]	9/2 — 76 7/2 — 33 5/2 — 0		0.25	5.1	28.3	a	45.3	e	29	32	18
Np^{237}	[523 5/2]	9/2 — 160 7/2 — 103 5/2 — 60		0.25	5.1	36.9	a	45.3	e	11	9	10
Np^{239}	[642 5/2]	7/2 — 31 5/2 — 0		0.26	5.3	26.6	a	44.7	i	33	29	17
Np^{239}	[523 5/2]	9/2 — 173 7/2 — 117 5/2 — 14		0.26	5.3	36.9	a	44.7	i	10	8	10
Am^{241}	[523 5/2]	7/2 — 42 5/2 — 0		0.27	5.4	36.0	a	42.9	e	10	8	9
Am^{243}	[523 5/2]	7/2 — 126 5/2 — 84		0.27	5.4	36.0	a	44.5	e	11	8	9

Assignments are rather uncertain and these values have not been plotted in figs. B-E.

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Uppsala 1958. Almqvist & Wiksells Boktryckeri AB